Ch2 Review Challenge 1 Questions Involving Quadratic Functions:

A quadratic polynomial $f(x) = x^2 + px + q$, with p and q real numbers, is said to be a *double-up polynomial* if it has two real roots, one of which is twice the other.

(a) If a double-up polynomial $f(x)$ has $p = -15$, determine the value of q.

(b) If $f(x)$ is a double-up polynomial with one of the roots equal to 4, determine all possible values of $p + q$.

(c) Determine all double-up polynomials for which $p + q = 9$.

A parabola has equation $y = (x - 3)^2 + 1$.

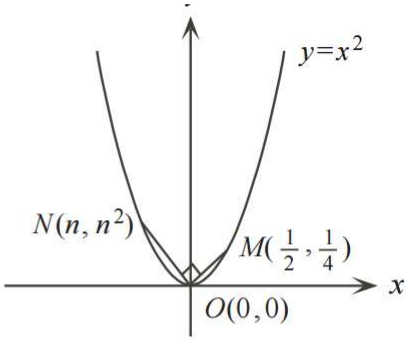
(a) What are the coordinates of the vertex of the parabola?

(b) A new parabola is created by translating the original parabola 3 units to the left and 3 units up. What is the equation of the translated parabola?

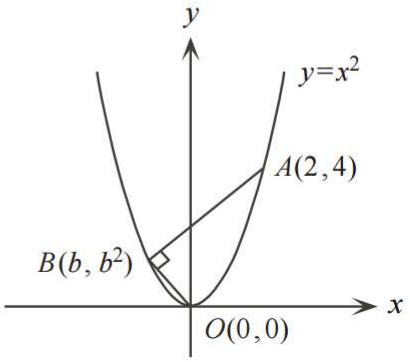
(c) Determine the coordinates of the point of intersection of these two parabolas.

(d) The parabola with equation $y = ax^2 + 4$, $a < 0$, touches the parabola with equation $y = (x - 3)^2 + 1$ at exactly one point. Determine the value of a.

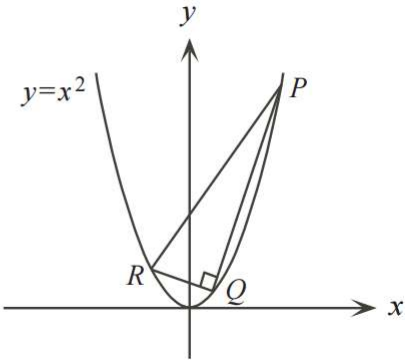
3. (a) Points $M(\frac{1}{2}, \frac{1}{4})$ and $N(n, n^2)$ lie on the parabola with equation $y = x^2$, as shown. Determine the value of n such that $\angle MON = 90°$.



(b) Points $A(2, 4)$ and $B(b, b^2)$ are the endpoints of a chord of the parabola with equation $y = x^2$, as shown. Determine the value of b so that $\angle ABO = 90°$.



(c) Right-angled triangle PQR is inscribed in the parabola with equation $y = x^2$, as shown. Points P, Q and R have coordinates $(p, p^2), (q, q^2)$ and (r, r^2), respectively. If p, q and r are integers, show that $2q + p + r = 0$.



4. Consider the quadratic function $f(x) = x^2 - 4x - 21$.

(a) Determine all values of x for which $f(x) = 0$ (that is, $x^2 - 4x - 21 = 0$).

(b) If s and t are different real numbers such that $s^2 - 4s - 21 = t^2 - 4t - 21$ (that is, $f(s) = f(t)$), determine the possible values of $s + t$. Explain how you obtained your answer.

(c) If a and b are different positive integers such that $(a^2 - 4a - 21) - (b^2 - 4b - 21) = 4$, determine all possible values of a and b. Explain how you obtained your answer.

Let $P(x)$ be a quadratic polynomial with real coefficients satisfying $x^2 - 2x + 2 \le P(x) \le 2x^2 - 4x + 3$ for all real numbers x, and suppose $P(11) = 181$. Find $P(16)$.

6 The graphs $y = 3(x-h)^2 + j$ and $y = 2(x-h)^2 + k$ have y-intercepts of 2013 and 2014, respectively, and each graph has two positive integer x-intercepts. Find h.

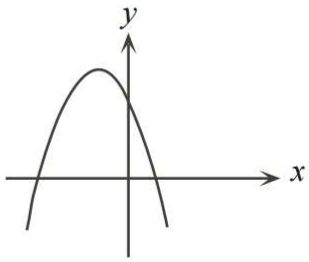
Fermat 2008

22. For how many integers k do the parabolas with equations $y = -\frac{1}{8}x^2+4$ and $y = x^2-k$ intersect on or above the x-axis?

(A) 9 **(B)** 32 **(C)** 33 **(D)** 36 **(E)** 37

Fermat 2007

20. The graph of the function $y = ax^2 + bx + c$ is shown in the diagram. Which of the following must be positive?



(A) a **(B)** bc **(C)** ab^2
(D) $b-c$ **(E)** $c-a$

Fermat 2014

25. Points $P(r,s)$ and $Q(t,u)$ are on the parabola with equation $y = x^2 - \frac{1}{5}mx + \frac{1}{5}n$ so that $PQ = 13$ and the slope of PQ is $\frac{12}{5}$. For how many pairs (m,n) of positive integers with $n \le 1000$ is $r+s+t+u = 27$?

(A) 28 **(B)** 26 **(C)** 27 **(D)** 29 **(E)** 25